RAȚIONALIZAREA NUMITORULUI UNEI FRACȚII

A raționaliza numitorul unei fracții ce conține radicali înseamna a transforma fracția prin amplificări sau simplificări cu scopul de a obține o fracție ce nu nu mai conține radicali la numitor.

Dacă fracția $\frac{F}{E}$ se amplifică cu C vom obține $\frac{F}{E}=\frac{FC}{EC}$.În cazul în care E conține radicali şi EC nu mai conține radicali, C se numeşte conjugata expresiei E.

Nr. crt.	Expresia E	Expresia conjugată a expresiei E
1.	$\sqrt{a}$	$\sqrt{a}$
2.	$\sqrt{a}\pm\sqrt{b}$	$\sqrt{a}\mp\sqrt{b}$
3.	$\sqrt[n]{a}$	$\sqrt[n]{a^{n-1}}$
4.	$a+\sqrt{b}$	$a-\sqrt{b}$
5.	$a-\sqrt{b}$	$a+\sqrt{b}$
6.	$a\pm\sqrt[3]{b}$	$a^2\mp a\sqrt[3]{b}+\sqrt[3]{b^2}$
7.	$\sqrt[3]{a}\pm\sqrt[3]{b}$	$\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2}$
8.	$\sqrt[n]{a}-\sqrt[n]{b}$	$\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}\cdot b}+\sqrt[n]{a^{n-3}\cdot b^2}+\ldots+\sqrt[n]{a^2\cdot b^{n-3}}+\sqrt[n]{ab^{n-2}}+\sqrt[n]{b^{n-1}}$
9.	$\sqrt[2n+1]{a}+\sqrt[2n+1]{b}$	$\sqrt[2n+1]{a^{2n}}-\sqrt[2n+1]{a^{2n-1}\cdot b}+\sqrt[2n+1]{{}^{2n-2}\cdot b^2}-\ldots+\sqrt[2n+1]{a^2b^{2n-2}}-\sqrt[2n+1]{ab^{2n-1}}+\sqrt[2n+1]{b^{2n}}$
10.	$\sqrt[n]{a}+\sqrt[n]{b}$	$\left(\sqrt[n]{a}-\sqrt[n]{b}\right)\left(\sqrt[n]{a^{2(n-1)}}+\sqrt[n]{a^{2(n-2)}b^2}+\ldots+\sqrt{a^2b^{2(n-2)}}+\sqrt[n]{b^{2(n-1)}}\right)$
11.	$\sqrt{a}+\sqrt{b}+\sqrt{c}$	$\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)\left(a+b-c-2\sqrt{ab}\right)$
12.	$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}$	$\left(\sqrt{a}+\sqrt{b}-\sqrt{c}-\sqrt{d}\right)\left(a+b-c-d-2\sqrt{ab}+2\sqrt{cd}\right)\cdot$ $\cdot\left[\left(a+b-c-d\right)^2-4ab-4cd-8\sqrt{abcd}\right]$
13.	$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$	$\left(\sqrt[3]{a^2}+\sqrt[3]{b^2}+\sqrt[3]{c^2}-\sqrt[3]{ab}-\sqrt[3]{bc}-\sqrt[3]{ca}\right)\cdot$ $\cdot\left[\left(a+b+c\right)^2+3\left(a+b+c\right)\sqrt[3]{abc}+9\sqrt[3]{a^2b^2c^2}\right]$

14.	$a^2\mp a\sqrt[3]{b}+\sqrt[3]{b^2}$	$a\pm\sqrt[3]{b}$
15.	$\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2}$	$\sqrt[3]{a}\pm\sqrt[3]{b}$
16.	$\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}\cdot b}+\sqrt[n]{a^{n-3}\cdot b^2}+\ldots+\sqrt[n]{a^2\cdot b^{n-3}}+\sqrt[n]{ab^{n-2}}+\sqrt[n]{b^{n-1}}$	$\sqrt[n]{a}-\sqrt[n]{b}$
17.	$\sqrt[2n+1]{a^{2n}}-\sqrt[2n+1]{a^{2n-1}\cdot b}+\sqrt[2n+1]{{}^{2n-2}\cdot b^2}-\ldots+\sqrt[2n+1]{a^2b^{2n-2}}-\sqrt[2n+1]{ab^{2n-1}}+\sqrt[2n+1]{b^{2n}}$	$\sqrt[2n+1]{a}+\sqrt[2n+1]{b}$
18.	$\left(\sqrt[n]{a}-\sqrt[n]{b}\right)\left(\sqrt[n]{a^{2(n-1)}}+\sqrt[n]{a^{2(n-2)}b^2}+\ldots+\sqrt[n]{a^2b^{2(n-2)}}+\sqrt[n]{b^{2(n-1)}}\right)$	$\sqrt[n]{a}+\sqrt[n]{b}$
19.	$\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)\left(a+b-c-2\sqrt{ab}\right)$	$\sqrt{a}+\sqrt{b}+\sqrt{c}$
20.	$\left(\sqrt{a}+\sqrt{b}-\sqrt{c}-\sqrt{d}\right)\left(a+b-c-d-2\sqrt{ab}+2\sqrt{cd}\right)\cdot$ $\cdot\left[\left(a+b-c-d\right)^2-4ab-4cd-8\sqrt{abcd}\right]$	$\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}$

21.	$\left(\sqrt[3]{a^2}+\sqrt[3]{b^2}+\sqrt[3]{c^2}-\sqrt[3]{ab}-\sqrt[3]{bc}-\sqrt[3]{ca}\right)\cdot$ $\cdot\left[(a+b+c)^2+3(a+b+c)\sqrt[3]{abc}+9\sqrt[3]{a^2b^2c^2}\right]$	$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$
22.	$\sqrt[n]{a^{n-1}}$	$\sqrt[n]{a}$
23.	$\sqrt[n]{a\pm\sqrt{b}}$	$\sqrt[n]{\left(a\mp\sqrt{b}\right)^{n-1}}$
24.	$\sqrt[n]{\sqrt{a}\pm\sqrt{b}}$	$\sqrt[n]{\left(\sqrt{a}\mp\sqrt{b}\right)^{n-1}}$
25.	$\sqrt[n]{a\pm\sqrt[3]{b}}$	$\sqrt[n]{\left(a^2\mp a\sqrt[3]{b}+\sqrt[3]{b^2}\right)^{n-1}}$
26.	$\sqrt[n]{\sqrt[3]{a}\pm\sqrt[3]{b}}$	$\sqrt[n]{\left(\sqrt[3]{a^2}\mp\sqrt[3]{ab}+\sqrt[3]{b^2}\right)^{n-1}}$
27.	$\sqrt[k]{\sqrt[n]{a}-\sqrt[n]{b}}$	$\sqrt[k]{\left(\sqrt[n]{a^{n-1}}+\sqrt[n]{a^{n-2}\cdot b}+\sqrt[n]{a^{n-3}\cdot b^2}+\ldots+\sqrt[n]{a^2\cdot b^{n-3}}+\sqrt[n]{ab^{n-2}}+\sqrt[n]{b^{n-1}}\right)^{k-1}}$
28.	$\sqrt[k]{\sqrt[2n+1]{a}+\sqrt[2n+1]{b}}$	$\sqrt[k]{\left(\sqrt[2n+1]{a^{2n}}-\sqrt[2n+1]{a^{2n-1}\cdot b}+\sqrt[2n+1]{{}^{2n-2}\cdot b^2}-\ldots+\sqrt[2n+1]{a^2b^{2n-2}}-\sqrt[2n+1]{ab^{2n-1}}+\sqrt[2n+1]{b^{2n}}\right)^{k-1}}$
29.	$\sqrt[k]{\sqrt[n]{a}+\sqrt[n]{b}}$	$\sqrt[k]{\left(\left(\sqrt[n]{a}-\sqrt[n]{b}\right)\left(\sqrt[n]{a^{2(n-1)}}+\sqrt[n]{a^{2(n-2)}b^2}+\ldots+\sqrt[n]{a^2b^{2(n-2)}}+\sqrt[n]{b^{2(n-1)}}\right)\right)^{k-1}}$
30.	$\sqrt[n]{\sqrt{a}+\sqrt{b}+\sqrt{c}}$	$\sqrt[n]{\left(\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)\left(a+b-c-2\sqrt{ab}\right)\right)^{n-1}}$

Test nr.14a
Autor:Ion Bursuc

1. Raționalizați numitorii următoarelor fracții:

a) $\frac{1}{\sqrt{2}}$; b) $\frac{2}{\sqrt[3]{2}}$; c) $\frac{5}{\sqrt{2}+\sqrt{3}}$; d) $\frac{1}{1+\sqrt{3}}$; e) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}$; f) $\frac{2}{\sqrt[3]{3}+1}$.

Soluție:

a) $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{\left(\sqrt{2}\right)^2}=\frac{\sqrt{2}}{2}$;

b) $\frac{2}{\sqrt[3]{2}}=\frac{2\sqrt[3]{2^2}}{\sqrt[3]{2\cdot 2^2}}=\frac{2\sqrt[3]{4}}{2}=\sqrt[3]{4}$;

c) $\frac{5}{\sqrt{2}+\sqrt{3}}=\frac{5\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}=\frac{5\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=5\sqrt{3}-5\sqrt{2}$;

d) $\frac{3}{1+\sqrt{3}}=\frac{3\left(1-\sqrt{3}\right)}{1^2-\left(\sqrt{3}\right)^2}=\frac{3\sqrt{3}-3}{2}$;

e) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}=\frac{\sqrt[3]{3^2}+\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}}{\left(\sqrt[3]{3}-\sqrt[3]{2}\right)\left(\sqrt[3]{3^2}+\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}\right)}=\frac{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}{\left(\sqrt[3]{3}\right)^3-\left(\sqrt[3]{2}\right)^3}=\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}$;

f) $\frac{2}{\sqrt[3]{3}+1}=\frac{2\left(\sqrt[3]{3^2}-\sqrt[3]{3}\cdot 1+1^2\right)}{\left(\sqrt[3]{3}+1\right)\left(\sqrt[3]{3^2}-\sqrt[3]{3}\cdot 1+1^2\right)}=\frac{2\sqrt[3]{9}-2\sqrt[3]{3}+2}{\left(\sqrt[3]{3}\right)^3+1^3}=\frac{2\sqrt[3]{9}-2\sqrt[3]{3}+2}{4}=\frac{\sqrt[3]{9}-\sqrt[3]{3}+1}{2}$.

2. Raționalizați numitorii următoarelor fracții:

a) $\frac{1}{\sqrt{2+\sqrt{3}}}$; b) $\frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}$; c) $\frac{2}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$; d) $\frac{1}{\sqrt{3}+\sqrt{2}+\sqrt{6}+\sqrt{7}}$.

Soluție:

a) $\frac{1}{\sqrt{2+\sqrt{3}}}=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}}=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2^2-\left(\sqrt{3}\right)^2}}=\sqrt{2-\sqrt{3}}$

b)

$$\frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}=\frac{1}{\sqrt[3]{3^2}+\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}}=\frac{\sqrt[3]{3}-\sqrt[3]{2}}{\left(\sqrt[3]{3^2}+\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}\right)\left(\sqrt[3]{3}-\sqrt[3]{2}\right)}=\frac{\sqrt[3]{3}-\sqrt[3]{2}}{\left(\sqrt[3]{3}\right)^3-\left(\sqrt[3]{2}\right)^3}=\sqrt[3]{3}-\sqrt[3]{2}$$

c) $\frac{2}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=\frac{2\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)}{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)}=\frac{2\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)}{\left(\sqrt{2}+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}=\frac{2\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)}{2\sqrt{6}}=$

$\frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{6}}=\frac{\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)\sqrt{6}}{6}$

d) $\frac{1}{\sqrt{3}+\sqrt{2}+\sqrt{6}+\sqrt{7}}=\frac{\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}}{\left(\sqrt{3}+\sqrt{6}+\sqrt{2}+\sqrt{7}\right)\left(\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}\right)}=\frac{\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}}{\left(\sqrt{3}+\sqrt{6}\right)^2-\left(\sqrt{2}+\sqrt{7}\right)^2}=$

$=\frac{\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}}{2\sqrt{18}-2\sqrt{14}}=\frac{\left(\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}\right)\left(\sqrt{18}-\sqrt{14}\right)}{2\sqrt{18}^2-2\sqrt{14}^2}=\frac{\left(\sqrt{3}+\sqrt{6}-\sqrt{2}-\sqrt{7}\right)\left(\sqrt{18}-\sqrt{14}\right)}{8}$.

Test nr.14b(Temă pentru acasă)
Autor:Ion Bursuc

1. Raționalizați numitorii următoarelor fracții:

a) $\frac{1}{\sqrt{3}}$; b) $\frac{2}{\sqrt[3]{3}}$; c) $\frac{5}{\sqrt{2}-\sqrt{3}}$; d) $\frac{1}{2-\sqrt{3}}$; e) $\frac{3}{\sqrt[3]{5}-\sqrt[3]{2}}$; f) $\frac{5}{\sqrt[3]{4}+1}$.

2. Raționalizați numitorii următoarelor fracții:

a) $\frac{1}{\sqrt{3+\sqrt{8}}}$; b) $\frac{1}{\sqrt[3]{16}+\sqrt[3]{12}+\sqrt[3]{9}}$; c) $\frac{2}{\sqrt{2}+\sqrt{11}+\sqrt{13}}$; d) $\frac{1}{\sqrt{3}+\sqrt{2}+\sqrt{10}+\sqrt{11}}$.